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Correlated quantum fluids and the search for a new theory of metals

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One of the key discoveries at the close of the last century was that of the electron (Nobel prize 1906). As well as heralding the dawn of particle physics, it also gave a fundamental basis for understanding electrical conduction in solids. As this century closes, we are discovering new materials in which the interactions between electrons can lead to a quantum fluid where the electron no longer appears to be the fundamental current-carrying entity. Notable examples are the high-temperature cuprate superconductors, heavy-fermion compounds and semiconductors in the quantum Hall regime. Here we review why a radical revision of our understanding of the electron quantum fluid is needed. A common thread is the possibility of exotic quantum phases of matter arising from strong electron correlations, reduced dimensionality, and the competition between magnetism and electronic conduction.

Keywords: metals; superconductivity; quantum Hall effect

1. Introduction

Looking back over 100 years in physics, the discovery of the electron and the birth of quantum physics are two milestones that underpin our microscopic understanding of the physical world. These ideas belonged, initially, to the realm of atomic physics, concerned with the behaviour of tens of electrons orbiting a point-like nucleus. However, it soon became evident that the behaviour of solids and liquids, which consist of 10^{23} or more electrons and nuclei, requires a new level of understanding that cannot be obtained from the study of isolated atoms or molecules. This is the domain of condensed-matter physics.

From the very beginning, condensed-matter physicists have searched for microscopic descriptions of everyday properties such as the cohesion of atoms in solids and the ability of metals to conduct heat and electricity. The discovery and fabrication of new materials (magnets, superconductors and liquid crystals, etc.) have continually challenged our understanding by consistently producing new types of behaviour. We have learnt that a reductionist viewpoint that seeks to identify the basic constituents and their interactions (here, the electrostatic forces between electrons and nuclei) is not sufficient to understand the properties of many condensed-matter systems. Instead, we find that every new material brings these fundamental constituents together in a different way and the whole is frequently greater than the

sum of the parts. This idea of emergent phenomena in complex systems is one of the main driving forces of modern condensed-matter physics.

These new condensed-matter phenomena are the *macroscopic* consequences of microscopic laws. They are every bit as fundamental as the basic constituents themselves in the sense that they could not have been guessed by an extrapolation of the physics of simpler systems. We should not be surprised to find new behaviour here. After all, the physics of individual atoms occurs at nanometre distances over femtosecond time intervals. These are very small scales compared with those found in macroscopic phenomena, such as the magnetic ordering of electrons over the entire length of an iron bar.

Understanding how new physical phenomena emerge from complex systems often requires radically new concepts. One dramatic realization of this forms the subject of this article. Here, we focus on metallic solids, where the interactions between the electrons can lead to strong correlations in their motion. As a consequence, we find exotic quantum phases of matter where *new* types of particles (such as fractionally charged objects) provide a much better basis for their description than that of the electron. We start by reviewing some of the basic concepts behind our current understanding. We will then describe recent discoveries that appear to demand a whole new conceptual framework for the theory of metals. Not only do these discoveries pose a challenge at a theoretical level, they also point to opportunities on a technological front.

2. The Fermi sea

Shortly after the discovery of the electron, it was realized that a metal should be viewed as a collection of atoms each of which has given up, on average, one to two electrons to be shared among the other atoms. These electrons are mobile, so they conduct electricity from one end of the metal to the other. They were first modelled as non-interacting particles moving in a crystalline lattice of ions. However, in contrast to a classical gas of particles (like neon), the small mass of the electron means that quantum mechanics plays a vital role; the electrons form a *quantum fluid*.

The electrons' contribution to the heat capacity shows how dramatic the role of quantum mechanics is. The heat capacity measures the incremental change in the energy of the electrons for a small rise in temperature. Classically, each electron has a thermal energy of $\frac{3}{2}k_{\mathrm{B}}T$, so that it contributes a temperature-independent heat capacity of $\frac{3}{2}k_{\mathrm{B}}$ (where k_{B} is the Boltzmann constant). However, experiments show that the heat capacity is much smaller, decreasing proportionally with the temperature T at low temperatures.

The explanation lies in the quantum nature of the electrons. Quantum theory teaches us that particles can also be described as waves (and vice versa). For a single electron, these are simply standing waves in the material. The momentum of the particle p is related to the wavelength λ by de Broglie's relation: $p = h/\lambda$ (h is Planck's constant). Only specific wavelengths are allowed if we are to fit the standing waves between the end walls of the solid. This *quantizes* the momentum: $p = nh/2L$, where n is a whole number and L is the separation of the walls. The system favours the configuration of minimum total energy, the 'ground state'. For one electron, this is the zero-momentum state. For many electrons, we have to take

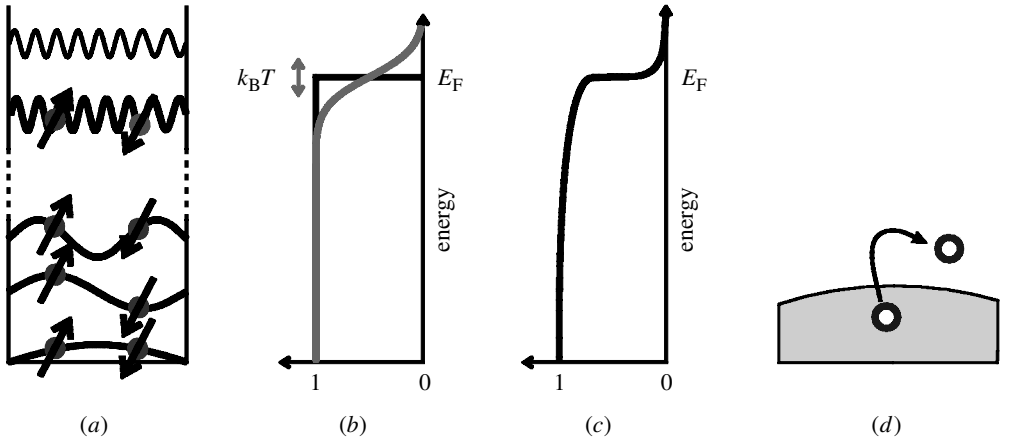

Figure 1. (*a*) Filling up the Fermi sea. (*b*) Electron distribution for a non-interacting Fermi sea (black line, $T = 0$; grey line, finite temperature). (*c*) Electron distribution at $T = 0$ with interactions. (*d*) Particle–hole excitation.

into account the Pauli exclusion principle, which states that only two electrons (with opposite magnetic orientations) can be in the same quantum state. If we assume that the electrons in a metal do not interact with each other, then it is easy to find out the ground-state configuration. Working upwards in energy, we fill up the available standing-wave states one by one with two electrons. To accommodate all the electrons, we will reach waves of very short wavelengths, i.e. high momenta p and energies (figure 1*a*). This top energy level is called the 'Fermi level', E_F, and this configuration for the electrons is called the 'Fermi sea'.

At zero temperature, the surface of this Fermi sea is sharp, in the sense that the standing-wave states are either occupied or not (figure 1*b*). Thermal effects can smear out this way of distributing the electrons among the quantum states. The crucial point is that they only disturb the distribution over an energy scale of k_BT, which is typically much smaller than E_F. More precisely, we can create a thermal 'excitation' by giving additional energy of not more than k_BT to an electron. Since the states below the Fermi level are filled up completely with particles, this process is only possible if the electron ends up with an energy *above* the Fermi level. This is called a 'particle–hole excitation' because it creates a vacancy below the Fermi sea by moving a particle from below to above the sea level (figure 1*d*). We see that only the standing-wave states within an energy interval of width k_BT around the Fermi level are affected by thermal effects.

We can now return to the question of the heat capacity. The classical theory assumes that each electron is affected by a temperature change in the same way. The Fermi sea described here behaves quite differently: only the electrons with energies not more than k_BT away from the Fermi level are affected. Therefore, the quantum result is smaller than the classical prediction by a factor of k_BT/E_F. This explains why the electronic heat capacity becomes smaller and smaller as the system is cooled down. Arguments similar to the ones outlined above have provided a quantitative understanding of many other basic properties of metals as a *quantum* electron fluid.

3. Fermi liquid theory: quasi-particles

The non-interacting Fermi sea is a remarkably simple model of metals. In fact, we might wonder how we could be so bold as to contemplate such a model at all. After all, we have treated the electrons as independent particles with no interactions (save the requirements of quantum mechanics), when we know that there are strong electrostatic forces between them!

This picture was put on a firm theoretical footing in the 1950s by Landau's Fermi liquid theory. Landau considered turning on the interactions between the electrons slowly and observing how the system evolved. He postulated that, for low-energy excitations (i.e. those involving states close to the Fermi level), there would be a one-to-one correspondence between the excitations of the interacting electron system with those of the non-interacting Fermi sea. In other words, we can still imagine excitations that resemble the particle–hole excitations described above, but the particles involved are modified from the original electrons. For instance, these 'quasi-particles' have a mass that is not the same as the electron mass, although it is usually still of the same order of magnitude. These quasi-particles are still electron-like in the sense that they carry the same electrical charge and magnetism as the original electron. Moreover, a sharp Fermi surface continues to exist at zero temperature (figure 1*c*).

The important difference between the quasi-particles and the original electrons is that the quasi-particles only interact very weakly, meaning that they scatter off of each other only infrequently (at low temperatures or close to the Fermi level). A qualitative reason for this is that when a negatively charged electron is added to the system, it expels other electrons from its vicinity, creating a positively charged cloud around it. This charge cloud compensates for the electrostatic influence of the added electron on other distant electrons. Thus, the electrostatic repulsion of the bare electron is moderated. We can regard the quasi-particle as a 'bare' electron with its associated charge cloud.

The concept of quasi-particles lies at the heart of Fermi liquid theory. It is important to note that they are not just theoretical constructs: they are the 'elementary particles' of condensed-matter physics since they determine all the physical properties of the system at low temperatures. This is because these properties, such as the heat capacity, only require the presence of a well-defined Fermi surface, and are not sensitive to whether the Fermi sea consists of electrons or quasi-particles.

To borrow a phrase from high-energy physics, Fermi liquid theory has become the 'standard model' of metals. It explains why a wide range of compounds made up of elements from different parts of the periodic table might behave in a similar way. However, it is by no means a 'grand unified theory'. There are many exceptions to the rule that point us towards novel phenomena. Indeed, they challenge us to find a new conceptual framework for the theory of metals.

4. Beyond Fermi liquids

The central postulate in Fermi liquid theory is that the ground state of the electron system is continuously related to the non-interacting Fermi sea if we introduce the electronic interactions slowly. This is the principle of 'adiabatic continuity'. The validity of this assumption depends on the strength and nature of the electronic interactions as well as the effective dimensionality of the system.

A clear case of the breakdown of Fermi liquid theory is when a metal fails to behave as an ordinary metal. For instance, some metals become superconducting below a well-defined critical temperature T_c (e.g. 7 K for Pb and 23 K for Nb_3Ge). Above T_c the material is quite ordinary. Below it there is no sign of electrical resistance at all. The material also becomes a strong diamagnet, expelling magnetic flux from its interior (the Meissner effect). When magnetic flux is forced into the material with a strong magnetic field, it enters as thin lines, each carrying a *quantized* amount of flux, $h/2e$, where h is Plank's constant. If we look at the electronic heat capacity, it is strongly suppressed, indicating that there are *no* particle-like or hole-like states near the Fermi level at all.

What has happened? Clearly, the ground state of the system has undergone some radical reorganization. In this case, the electrons (or, more precisely, the electron-like quasi-particles) have paired up. The origin of the electron pairing was explained by Bardeen *et al.* (1957) as the result of an attractive interaction mediated by the sound waves in the solid. It is one of the most successful theories in condensed-matter physics.

The new entities (Cooper pairs) belong to a different class of particles from electrons. Like photons, they are not subject to the Pauli exclusion principle. Instead, they follow the Bose–Einstein distribution where they *prefer* to occupy the same quantum state. They behave as if they are one single quantum entity, and so are immune from the random scattering of individual charge carriers, which causes resistance in normal metals. The analogous phenomenon for photons is the laser.

In this example, the loss of adiabatic continuity with the Fermi liquid state is signalled by a phase transition to a new state of matter (like the freezing of water to ice). Other examples exist, such as ferromagnetism (e.g. in Fe below 1043 K), where the electrons align their magnetic directions spontaneously without the need for an external magnetic field. However, much of the recent interest is in materials in which we see a metallic state very different from the one that Landau envisaged and apparently not separated from it by a phase transition. Among them, the materials with the greatest technological importance are arguably the cuprate superconductors, in which a strange metallic state is observed before one cools to the superconducting transition.

5. The mystery of the cuprate superconductors

The cuprates encompass almost 30 distinct crystalline structures and contain upwards of three different elements. Whereas the onset of superconductivity typically occurs at tens of kelvins for other materials, the cuprates have critical temperatures of the order of 100 K (e.g. 90 K for $YBa_2Cu_3O_7$). The practical significance is that this is above the boiling point of nitrogen (77 K), and so inexpensive liquid nitrogen can be used as a refrigerant instead of liquid helium for other materials. Moreover, the existence of the cuprates raises hopes for the discovery of room-temperature superconductors. This will remain a major driving force behind superconductivity research in physics, chemistry and material science.

Despite a decade of intensive research, we still do not have a good theoretical understanding of the cuprates. This is because there are competing interactions, which influence the physics of the electron system. However, we are coming to the consensus that the answer must lie in a common feature of all the cuprates; a layered

structure of CuO_2 planes (figure 2). In each of these planes, the copper atoms are arranged in the sites of a square lattice. The oxygen atoms sit between neighbouring copper sites. In their pristine state (e.g. La_2CuO_4), each copper atom donates one electron as a charge carrier. A simple model would, therefore, consider electrons hopping from site to site on a square lattice. The two-dimensional nature of this motion is believed to play an important role in the behaviour of this system. This has, in part, prompted a search for unusual materials in low dimensions.

A non-interacting electron theory would predict that the system has a metallic Fermi sea. However, La_2CuO_4 is an insulator! The reason for this is that the repulsion between electrons is important here, and this has caused an electronic traffic jam. We have one electron per copper site, and an electron cannot hop to a neighbouring site because of the repulsion from the electron that is already there. This kind of insulating behaviour is called a Mott insulator. Mott insulators are expected to be antiferromagnets; the magnetic moment on copper alternates in direction as one moves from one site to its neighbour. This is observed in all the cuprates.

To make metals out of these insulators we have to relieve the traffic jam. This is done by changing the chemical composition of the compound so that electrons are transferred from the copper-oxide layers to atoms that reside between the layers. For La_2CuO_4 we can dope with Sr, which substitutes for La. In this way, electron vacancies (holes) are created in the layers, and electrons can now move by hopping into the vacancies. Another way of looking at this is to watch the vacancies being shunted around the lattice by the motion of the electrons. The holes have a positive charge relative to the undoped compound and so we can attribute the conductivity to mobile, positively charged holes.

As we increase the doping, antiferromagnetism soon disappears and the material becomes a metallic conductor that is also superconducting at low temperatures (figure 2). The critical temperature reaches a maximum optimal value of *ca.* 100 K when the lattice contains 15–25% vacancies. One might argue, therefore, that antiferromagnetism and superconductivity are competing effects here. On the other hand, the superconductivity is intimately related to the proximity of the conductor to a magnetic state; the superconductivity disappears at higher doping.

The superconducting state is unconventional in the cuprates, but the metallic state is itself unusual. One of the earliest observed anomalies is the linear proportionality between the resistance for currents in the CuO planes and the temperature. This is a robust phenomenon that occurs for nearly all the cuprates when the doping is optimal, and it holds over a wide range of temperatures, from T_c to 1000 K. Moreover, the origin of this resistance apparently involves only the electrons. This is surprising because no known theory predicts this behaviour from electronic scattering alone. In fact, Fermi liquid theory expects the resistance to be proportional to the *square* of the temperature through the scattering of one quasi-particle by another.

We can also measure the scattering rates of quasi-particles by observing the sideways current generated by the Lorentz force on the electrons due to a magnetic field (the Hall effect). Curiously, the resistance for these currents *does* give a scattering rate proportional to T^2. How can a single quasi-particle have two radically different scattering rates? The answers to this question and many other peculiarities of the cuprates remain highly controversial. Even the co-authors here have different proposals!

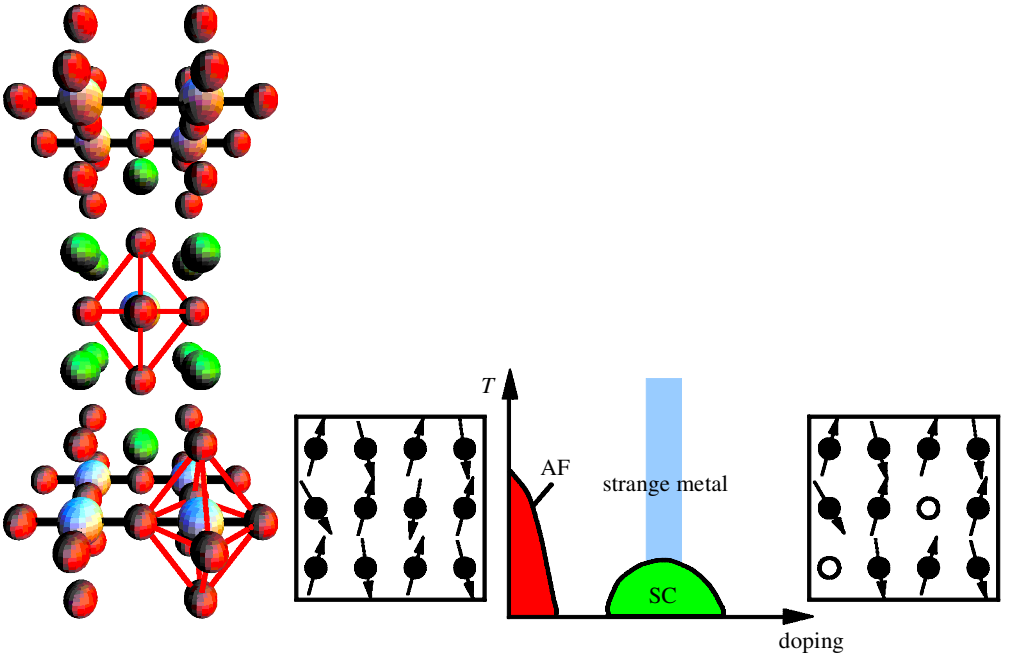

Figure 2. Structure of La_2CuO_4 (Cu, blue; O, red). Generic phase diagram of a cuprate superconductor. AF denotes an antiferromagnet and SC a superconductor.

As hinted at above, a theoretical model would have to give a quantitative description of how the mobility of the electronic charge and the antiferromagnetic correlations of the electronic magnetic moments are intimately related to each other. Unfortunately, even the simplest theoretical models that include these effects are mathematically intractable beyond one dimension. At present, it is also unclear how such a model would give rise to a mechanism for superconductivity.

Nevertheless, many theorists are coming to the view that the experimental anomalies point to a total breakdown of the Fermi liquid in the metallic state of the cuprates. In other words, the fundamentally current-carrying object might be radically different from the electron or any electron-like quasi-particle. Moreover, it appears that an ingredient essential to this breakdown of the Fermi liquid is the presence of some magnetic correlations in the system.

One fruitful direction of research is to broaden our perspective to see if a larger range of magnetic materials exhibits similar non-Fermi-liquid behaviour. In fact, the study of the magnetic metals and insulators predates the discovery of the cuprate superconductors. These compounds exhibit a wide range of behaviour, such as ferromagnetism, antiferromagnetism and superconductivity. For instance, $CePd_2Si_2$ is an antiferromagnetic metal at ambient pressure. The antiferromagnetism can, however, be destroyed at high pressures (28 000 atm). Tantalizingly, this material becomes superconducting upon the disappearance of the magnetic order (see figure 3). The superconducting critical temperature is very low (0.4 K), but the apparent competition between antiferromagnetism and superconductivity is qualitatively similar to the cuprates. Therefore, this compound might shed some light on how the Fermi-liquid state evolves towards a transition where a new quantum state of matter emerges. In

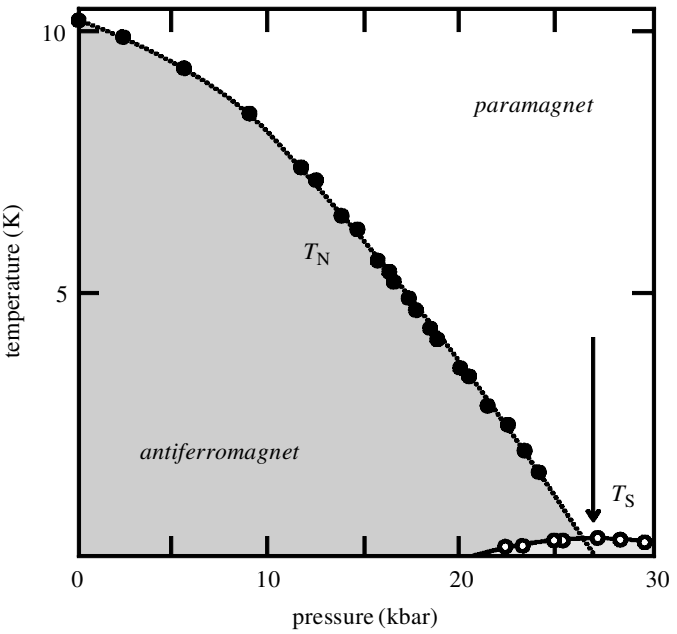

Figure 3. Phase diagram of $CePd_2Si_2$ (Julian *et al.* 1996). Under pressure, the magnetism may be 'squeezed out' of this material, turning it into an unconventional metal and a low-temperature superconductor.

particular, $CePd_2Si_2$ belongs to a class of materials known as 'heavy-fermion compounds', where a Fermi-liquid interpretation would deduce a quasi-particle that has a mass several orders of magnitude higher than the electron mass. We can regard this as a signal that Fermi liquid theory has been stretched to the limits of its applicability in these materials. The elucidation of the actual breakdown of the Fermi liquid state remains a challenge for the future.

6. Novel quasi-particles?

If we do not have a Fermi liquid, we need to look for a new way of characterizing the system. Does this mean that the system is now so complex that it defies description by any simple physical picture? Our hope is that even complex systems have organized ground states, and that these quantum states of matter can be characterized by novel quasi-particles. We describe below a few examples where non-electron-like quasi-particles are known to exist. Notice that, in addition to strong interactions, all these systems involve physics in an effectively reduced number of spatial dimensions.

(*a*) *Spin–charge separation*

One conjecture, put forward by Anderson (1997), for the metallic state of the cuprates is that charge and spin (i.e. magnetic moment) have somehow become decoupled in this system. Whereas charge and spin were tied together on the same object (the electron or the electron-like quasi-particle) in Fermi liquids, Anderson proposed that there are two separate quasi-particles, one for charge ('holon') and one for spin

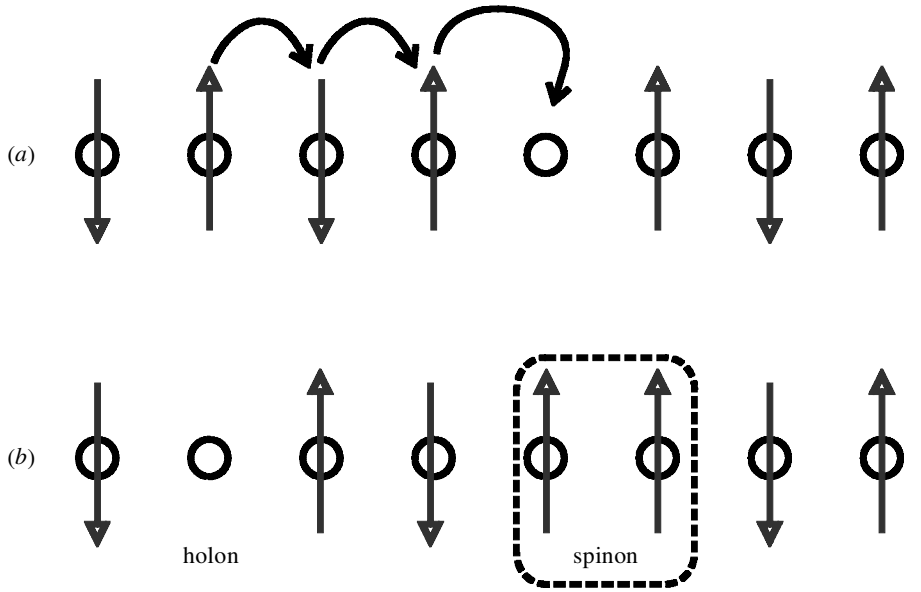

Figure 4. Spin–charge separation in one dimension.

('spinon'), and that an added electron 'falls apart' into these two objects! More precisely, the electrons in the system have reorganized themselves in such a way that charge and spin disturbances can become widely separated in space.

There is evidence of this spin–charge separation in the one-dimensional analogue of the cuprates. Consider a chain with one electron per site on the chain. As in the undoped cuprates, electrostatic repulsion makes this an antiferromagnetic Mott insulator. So, the spin direction of each electron is antiparallel to its neighbours. Suppose we remove a spin-down electron from this chain (figure 4*a*). Now, we can move the electron on the left of the vacancy onto this site, i.e. the charge disturbance (holon) moves to the left. Note that we have created a defect in the antiferromagnetic spin alignment, i.e. there are two neighbouring electrons with *parallel* spins. This is a spin disturbance (spinon) relative to the original state. If we reiterate this procedure with the new vacancy, we can move the charge disturbance (holon) far away from the defect in the spin arrangement (spinon) (figure 4*b*).

This illustration of 'spin–charge separation' can be formalized into a quantitative theory in one dimension. This has led to the search for quasi-one-dimensional compounds that might exhibit this behaviour. $SrCuO_2$ is a promising candidate with CuO chains (instead of planes), which is a Mott insulator in its pristine state. Although the two-dimensional problem relevant to the cuprates has no mathematical solution so far, our progress with their one-dimensional relatives is encouraging.

(*b*) *Fractional charges*

Other condensed-matter systems also exhibit novel coupling between spin and charge, where, by contrast with the electron, different proportions of electric charge and magnetism are bound together in the elementary excitations. An example can be found in semiconductors in high magnetic fields. Semiconductor devices, such as the field-effect transistor, make use of electrons confined at the interface between two

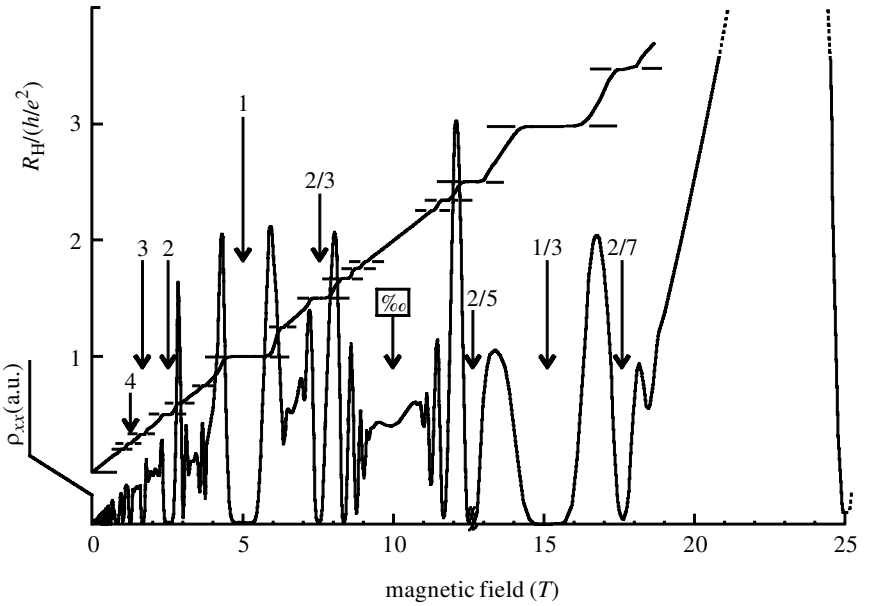

Figure 5. A schematic of a quantum Hall effect measurement, showing a plateau at $R_{\mathrm{H}} = h/\nu e^2$, for $\nu = 3, 2, 1, \frac{2}{3}, \ldots, \frac{2}{7}$ from left to right. The longitudinal resistance ρ_{xx} also displays sharp features (Tsui *et al.* 1982).

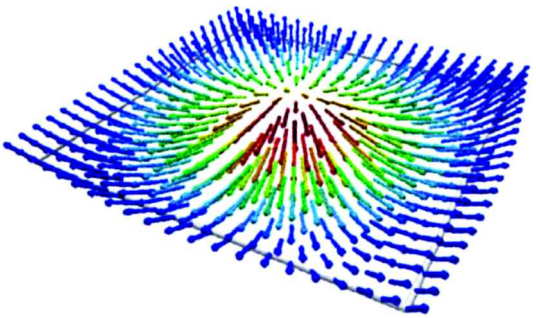

Figure 6. Spin configuration of a skyrmion caused by a reversed spin at the centre. All the spins point upwards far away. The central spin point downwards, polarizing its neighbours. (Courtesy of N. R. Cooper.)

semiconductors. This is, effectively, a two-dimensional sheet, so that reduced spatial dimension again comes into play. When an external magnetic field B is applied perpendicular to the plane of the electron gas, any current flow, I, generates a sideways Hall voltage V_{H}. The Hall voltage and the current are proportional to each other, and so we can define a Hall resistance: $R_{\mathrm{H}} = V_{\mathrm{H}}/I$.

For small magnetic fields, R_{H} increases smoothly with the magnetic field. However, at strong fields, its evolution develops steps and plateaux; it has become *quantized* at particular fractions of h/e^2, i.e. $R_{\mathrm{H}} = h/\nu e^2$ for $\nu = 1, 2, 3, \ldots$. This is the 'integer quantum Hall effect'. This quantization is so accurate and robust to extraneous effects, such as disorder, that it has become the metrological standard for electrical resistance. For very clean systems and even lower temperatures, we observe a

'fractional quantum Hall effect' as well with an additional plateau of R_{H} at odd-denominator fractions, e.g. $h/\nu e^2$ for $\nu = \frac{2}{3}, \frac{2}{5}, \frac{1}{3}$ (see figure 5).

The discovery and exposition of the fractional effect won Laughlin, Störmer and Tsui the 1998 Nobel prize. The most spectacular property of this system is that charge excitations come in fractional units of the elementary charge. When the system has an R_{H} plateau at $3h/e^2$, the quasi-particle appears to carry charge $e/3$. (The spins of the electrons are completely aligned with the strong magnetic field so that the excitation carries no magnetism.)

Clearly, the electrons have organized themselves into a novel ground state. Here, the electronic motion is highly correlated with electrons circulating around each other in small groups of three, continuously changing from one group to the next. Interestingly, the idea of magnetic flux quanta (in units of h/e) from superconductivity theory enters this story as well. For the $\nu = \frac{1}{3}$ plateau to occur, the magnetic flux density has to be near 3 flux quanta per electron. One particular theoretical view of this state interprets each electron as having combined with 3 flux quanta (see Lee & Zhang 1996). The new composite particle now obeys Bose–Einstein statistics and is able to condense into a state resembling a superconductive state. Now, an elementary excitation of this system is the introduction of a unit flux quantum, which, from the flux per electron counting, corresponds only to one-third of the electronic charge. Again, it should be emphasized that these fractional charges are not theoretical constructs. Indeed, they have been observed directly in recent experiments.

(*c*) *Skyrmions*

The integer quantum Hall state also has interesting quasi-particles. In the previous section, we have assumed that all the electron spins are aligned with the external magnetic field. This is the Zeeman effect. Recent experiments have shown that the Zeeman effect in a solid is tunable by applying pressure to the solid. In particular, it gets weaker so that spin reversals are now possible.

It turns out that, even in the absence of an aligning field, the electrons in the quantum Hall regime prefer to align their spins parallel to each other. In other words, it is an intrinsic ferromagnet, like an iron bar. Consider adding an electron to this ferromagnet. In the integer Hall state, all the quantum states of one spin state have been occupied, and the Pauli exclusion principle forces the additional electron to occupy a state with its spin antiparallel to its neighbours. The inherent ferromagnetism of this electron gas does not favour this misalignment and so the neighbouring electrons attempt to rotate their spins to smooth out the disturbance. This may involve up to 20 spins around the new electron (figure 6). We see that adding one electron to the quantum Hall system results in the addition of one unit of charge e but *many* units of spin. This large-spin quasi-particle is called a 'skyrmion' (Sondhi *et al.* 1993), named after Skyrme who originally discussed such objects in the context of topological field theories in high-energy physics. This skyrmion has finally been observed in the completely different arena of condensed-matter physics.

7. Speculations

One of the attractions of condensed-matter physics is that theoretical progress has implications for applications as well. Our understanding of electrons in solids, based

on Landau's quasi-particle picture, underpins the silicon-based semiconductor technology that has transformed our lives in this century.

The discovery of high-temperature superconductors by Bednorz & Müller (1986) has already opened up new opportunities in superconductivity technology that were previously ruled out because of the expense and inconvenience of liquid-helium cryogenics. The absence of any electrical resistance allows the storage and transmission of electricity with no losses. The manufacture of superconducting cables is becoming a commercial possibility. Large magnets using superconductive coils are in use today in a wide range of applications, from particle accelerators to medical magnetic resonance imaging. The Meissner effect of magnetic flux expulsion can also be exploited to achieve magnetic levitation, leading to the prospect of frictionless trains.

On a smaller scale, superconductor electronics are also in sight. The cuprate superconductors are already in use in superconducting quantum interference devices (SQUIDs), which can detect minute changes in magnetic fields. These devices have a wide range of applications: from medical instruments to non-invasive testing for fracture in aircraft wheels.

The interplay of magnetism and electrical conduction has also found recent application in magnetic storage devices, such as computer hard disks. Here, one uses materials whose electrical resistance is strongly affected by a magnetic field (showing 'giant magnetoresistance'). There are probably many such possibilities using strongly correlated materials, where, as we have seen, the competition between magnetism and electrical conduction can play an important role. It is, for example, interesting to note that manganate relatives of the cuprates have even higher magnetoresistance (so-called 'colossal magnetoresistance').

Looking into the longer-term future, one might anticipate that a deeper understanding of the new types of exotic excitations arising in strongly correlated materials may lead to technologies every bit as rich as the single electron-quasi-particle physics that dominates the semiconductor device industry at present. Indeed, if technological dreams such as quantum computers are to be realized in practice, we will need to exploit the macroscopic consequences of quantum mechanics, which routinely dominates the physics of materials of current theoretical interest.

8. Conclusion

In conclusion, just as the turn of the last century saw the discovery of the electron and the key to understanding simple metals in terms of the electron quantum fluid, we are now seeing an increasing need for more exotic quantum fluids to account for the behaviour of more complex metallic states. Already we have seen materials in which the immutable electron should actually be viewed as separating into its magnetic and charged components. Also, fractionally charged objects have been identified in semiconductor devices and the dreams of particle theorists are being realized in condensed-matter systems.

These exciting developments are fuelled by the ever-increasing complexity of structures that material scientists can produce. From high-temperature superconductors to colossal magnetoresistance in manganates, we are continually surprised by the new phenomena that nature throws at us as we explore more complex systems. The radical new ideas required to understand such phenomena remain the challenge facing

the condensed-matter physicist in the next century, while technological application is surely only a matter of time.

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Derek Lee is a condensed-matter theorist. His research is directed at understanding quantum phases of matter whose existence depends on quantum correlations, interactions and disorder. These include disordered superfluids, dirty metals and the cuprate superconductors.

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